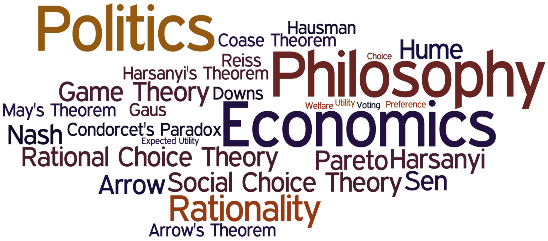
Methods in Philosophy, Politics and Economics: Individual and Group Decision Making

Eric Pacuit
University of Maryland
pacuit.org



Maximal Element

Suppose that $R \subseteq X \times X$ is a relation on X and $Y \subseteq X$.

$x \in Y$ is a **maximal** element of Y provided that there is no $z \in Y$ such that $z \neq x$, $z\ R\ x$, and not-$x\ R\ z$.

$x \in Y$ is **maximal** if there is no other element of Y that is *strictly* R-related to x.

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Suppose that $X = \{a, b, c, d\}$. Consider the following relations on X:

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That is, $y \in Y$ is maximal for a decision maker with rational preference (P, I) if there is no other element of Y that is strictly preferred to y.

Rational Choice

Suppose that X is set and $A \subseteq X$, and that (P, I) is a rational preference on X representing a decision maker's preferences.

$x \in A$ is a **rational choice** for the decision maker if x is a maximal element of A with respect to (P, I).

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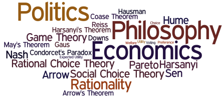

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Property β: Suppose that $A \subseteq X$ and $B \subseteq X$ and that x, y are in both A and B. If x and y are both maximal in A for (P, I) and x is maxinal in B for (P, I), then y is maximal B for (P, I).